

Teacher key: Same average rate, different functions

Private original formative diagnostic. Suggested time: 12 minutes; timing has not been observed. The key is local feedback guidance, not an official AP scoring rubric or score predictor.

The authoritative model is $f(x) = x^2 + 2x + 5$, $g(x) = 8x + 5$, with real inputs $0 \leq x \leq 6$. The formulas and supplied graph are correct. Only the f-table entry at $x = 2$ is deliberately wrong. The learner table does not mark its location. Later calculations must use the repaired table.

A. Audit and repair

The incorrect entry is $f(2) = 15$. The unique allowed repair is $f(2) = 13$:

$$f(2) = 2^2 + 2(2) + 5 = 4 + 4 + 5 = 13.$$

Graph 1 confirms the point **(2, 13) on f**. The g-point at the same input is **(2, 21)**; do not transfer that value into f's column. The table is the representation permitted to change. Altering a formula or moving a graph point does not satisfy the task's stated authority.

The complete corrected table is:

x	f(x)	g(x)
0	5	5
2	13	21
4	29	37
6	53	53

Other entries independently agree with their formulas: $f(0) = 5$, $f(4) = 16 + 8 + 5 = 29$, $f(6) = 36 + 12 + 5 = 53$; $g(0) = 5$, $g(2) = 16 + 5 = 21$, $g(4) = 32 + 5 = 37$, and $g(6) = 48 + 5 = 53$. No other numerical repair is allowed or needed.

Accept an equivalent calculation and an accurately identified graph coordinate. A response that gives 13 without function/input identification, formula work or graph evidence needs the missing evidence. If a learner changes $g(2)$ instead, ask them to trace the g-curve's label before comparing values. If $f(2)$ is evaluated as 11, ask whether the coefficient 2 is multiplying the input as well as the input being squared.

B. Equal-interval rates and their changes

All adjacent input intervals have width **2**. Divide the output change by 2:

Input interval	Calculation for f	Average rate for f	Calculation for g	Average rate for g
[0, 2]	$(13 - 5) \div (2 - 0) = 8 \div 2$	4	$(21 - 5) \div (2 - 0) = 16 \div 2$	8
[2, 4]	$(29 - 13) \div (4 - 2) = 16 \div 2$	8	$(37 - 21) \div (4 - 2) = 16 \div 2$	8
[4, 6]	$(53 - 29) \div (6 - 4) = 24 \div 2$	12	$(53 - 37) \div (6 - 4) = 16 \div 2$	8

The quadratic's average rates are **4, 8, 12**. Each next rate is **4 greater**: $8 - 4 = 4$ and $12 - 8 = 4$. The linear function's rates are **8, 8, 8**; their change is **0** each time. Equal input widths make these successive comparisons comparable. An output difference alone is not an average rate; the denominator must account for the interval's input change.

Accept “f's average rate increases by 4 per successive 2-unit interval; g's stays at 8” or an equivalent numerically supported explanation. Calling f's **rate** “4 throughout” confuses the change between rates with a rate itself. Calling g's **rate** 0 confuses its unchanged rate with zero change in output.

Teacher verification, not an extra timed task: for f and an interval of width $h > 0$,

$$(f(x + h) - f(x)) \div h = \mathbf{2x + h + 2}.$$

At $h = 2$ this is $2x + 4$, giving 4, 8, 12 at interval starts 0, 2, 4. For g the average rate is 8 on every nonzero-width interval. For consecutive starts separated by 2, f's rates differ by 4. If a learner voluntarily normalizes that difference, **$4 \div 2 = 2$ per input unit** is also correct when labeled; it is not the requested raw successive-rate difference of 4. Physical units are not assigned in this mathematical model. In a dimensional model the rate has output/input units, and its change per input unit would have output/input² units.

Error routing:

- **f-rates 8, 16, 24 or g-rates 16**: these are output differences. Ask the learner to write $b - a$ below each one.

- **f-rates 5, 7, 12**: these follow the unrepaired $f(2) = 15$. Return to Part A and then recalculate both intervals touching $x = 2$.

- **Negative rates from increasing values**: check that output and input differences use the same endpoint order. Reversing both is acceptable; reversing only one changes the sign incorrectly.

- **Quadratic family inferred solely from three rates**: remind the learner that this activity supplies the family through its authoritative formula. Observed patterns alone do not fix every unobserved value.

- **Instantaneous-rate language**: ask which two endpoints were used. These are interval average rates; no derivative or rate at an individual point was calculated.

C. Equal whole-interval rates do not establish the same function

For f:

$$(f(6) - f(0)) \div (6 - 0) = (53 - 5) \div 6 = 48 \div 6 = \mathbf{8}.$$

For g:

$$(g(6) - g(0)) \div (6 - 0) = (53 - 5) \div 6 = 48 \div 6 = \mathbf{8}.$$

The equal average rates are correct. The conclusion that the functions are identical is **not justified**. An interior counterexample is $x = 2$, where $f(2) = 13$ and $g(2) = 21$. Another is $x = 4$, where 29 is not equal to 37. Any correctly evaluated interior input showing unequal outputs is acceptable. The whole-interval average uses the endpoint change and interval width; equal endpoint information does not force all intermediate values to agree.

Accept formula reasoning as well: $f(x) - g(x) = x^2 - 6x = x(x - 6)$, which is negative for $0 < x < 6$ and zero at the two endpoints. A learner need not factor or establish the entire interior inequality; a single valid interior counterexample disproves the same-function claim. Saying only “one is curved” needs a specific value or a supported mathematical explanation.

For the separate **table-alone** question, the answer is **no**. Four rows specify four input values; they do not determine every other real input in $[0, 6]$. Other functions can match those rows and differ between them. In the actual activity the provided formulas and declared families settle what f and g are; the learner is not asked to reject those givens.

Teacher-only existence example, not required from learners: adding a nonzero multiple of $x(x - 2)(x - 4)(x - 6)$ to either formula preserves all four table values while changing other values and the function family. This validates the finite-data limitation without adding a higher-degree task to the diagnostic.

If a learner reports the whole average as 48, ask which input change corresponds to those endpoints. If they infer that same endpoint values imply the same graph, ask for $f(2)$ and $g(2)$. If they say the supplied f might not be quadratic, separate the hypothetical absence of a formula from the actual stipulated formula.

Feedback, pacing and scope

Use these local checks to select one focused revision:

- A identifies and repairs the single entry, with both formula and graph evidence.
- B gives all six interval rates, shows a complete calculation for each function and distinguishes rates from output differences.
- B explains the changes of +4 and 0 across equal-width intervals without confusing them with the rates.
- C computes both whole-interval averages as 8 and rejects the same-function conclusion with a valid interior counterexample.
- C distinguishes the table-alone inference limit from the actual authoritative model.

Suggested 12-minute use: 1 minute to read the model and authority instructions; 2 minutes for A; 5 minutes for B; 4 minutes for C. This is a desk allocation. Allow more time or split feedback if learners need support with substitution, graph reading or fractions. Learner timing, usefulness and proficiency outcomes have not been observed.

Framework fit: current AP Precalculus CED Topic **1.3**, objectives **1.3.A/B** for interval rates and their changes; practices **2.A/2.B** for actual cross-representation reading/repair and **3.B/3.C** for numerical application and justified conclusions. Secant-slope language is background rather than a separately assessed construction here. No Topic 1.2/1.13, derivative, full-course coverage or official AP scoring claim is made. Exact item tags remain subject to the framework owner's independent review.

The numbers, tasks and key are original synthetic mathematics created after the root's overlap GO. Existing non-AP PreCalculus games already teach formulas, graph reasoning, numerical verification and correction. The bounded distinction is this prepared integrated diagnostic, not missing mathematical topics. Release remains held pending independent mathematics, item-alignment, print/render/accessibility and owner review.