

Same average rate, different functions

Goal: check representations and justify a conclusion using average rates.

Estimated time: 12 minutes. Input numbers and function values are dimensionless. A rate measures change in output per unit of input.

Given model

For every real input in $0 \leq x \leq 6$, the functions are defined by these rules:

Quadratic: $f(x) = x^2 + 2x + 5$

Linear: $g(x) = 8x + 5$

The formulas and **Graph 1** are correct. Exactly **one numerical entry in Table 1 is wrong**; all its other entries are correct. Repair only that table entry. Keep the formulas and graph unchanged.
Use the repaired table for Parts B and C.

Graph 1: correct graphs of the given functions

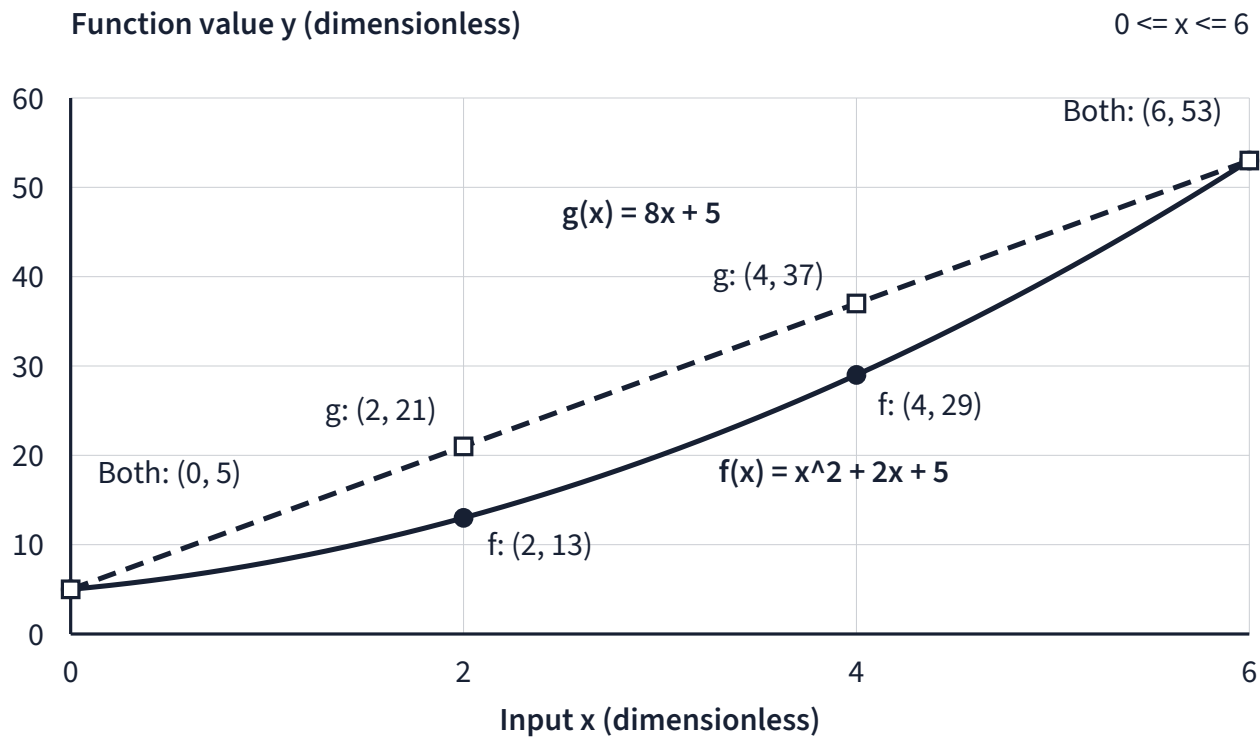


Table 1

x	f(x)	g(x)
0	5	5
2	15	21
4	29	37
6	53	53

A. Audit and repair

Identify the function and input of the wrong entry, then write its correct value. Show a calculation from the appropriate formula **and** cite a coordinate from Graph 1 that confirms your repair.

B. Compare equal-interval rates

For a function h , the average rate of change on $[a, b]$ is:

$(h(b) - h(a)) \div (b - a)$

Using the repaired table, complete the rates below. Show one full calculation for **each** function, including the input difference.

Input interval	Average rate for f	Average rate for g
$[0, 2]$	_____	_____
$[2, 4]$	_____	_____
$[4, 6]$	_____	_____

Describe how each function's average rates change from one interval to the next. Give the numerical change for each pattern. Explain why comparing these changes is meaningful for the equal-width intervals used here.

C. Test a conclusion

A reviewer claims: “**The functions have the same average rate on $[0, 6]$, so they are the same function throughout the domain.**”

Calculate each function's average rate on $[0, 6]$. Then decide whether the conclusion is justified. Use one interior input as a counterexample if needed, and explain why endpoint information does or does not settle the claim.

Now consider a separate question: if the formulas and declared families had **not** been supplied, would the four table rows alone prove a function is linear or quadratic for **every** input in $[0, 6]$? Explain briefly. The supplied formulas remain authoritative in the actual model.

Original formative practice. This is not an official AP question or scoring guide.